

HARD-PHOTON EMISSION IN $e^+e^- \rightarrow \bar{f}f$ WITH REALISTIC CUTS

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We derive compact analytical formulae of the Bonneau-Martin type for the reaction $e^+e^- \rightarrow \bar{f}f\gamma$ with cuts on minimal energy and acollinearity of the fermions, where the photons may be emitted both from the initial or final states. Soft-photon exponentiation is also taken into account.

One of the cleanest scattering processes at elementary particle accelerators is fermion-pair production in e^+e^- annihilation, potentially accompanied by one or few photons:

$$e^+e^- \rightarrow \bar{f}f + (n)\gamma. \quad (1)$$

Initial-state corrections may be written as an integral over the (normalized) invariant mass squared $R = s'/s$ of the final-state fermion pair:

$$\sigma_T^{ini}(s) = \int dR \sigma^0(s') \rho_T^{ini}(R), \quad (2)$$

where $\sigma^0(s')$ is an effective Born cross-section. The radiator function $\rho_T^{ini}(R)$ for the initial-state first-order corrections to the total cross-section σ_T , with soft-photon exponentiation, is [1]:

$$\rho_T^{ini}(R) = (1 + \bar{S}) \beta (1 - R)^{\beta-1} + \bar{H}_T^{ini}(R), \quad (3)$$

with

$$\bar{S} = \frac{3}{4}\beta + \frac{\alpha}{\pi}Q_e^2 \left(\frac{\pi^2}{3} - \frac{1}{2} \right) + h.o., \quad \beta = \frac{2\alpha}{\pi}Q_e^2 \left(\ln \frac{s}{m_e^2} - 1 \right), \quad (4)$$

and

$$\bar{H}_T^{ini}(R) = \left[H_{BM}(R) - \frac{\beta}{1-R} \right] + h.o., \quad (5)$$

where $h.o.$ stands for higher orders, and [2]:

$$H_{BM}(R) = \frac{1}{2} \frac{1+R^2}{1-R} \beta. \quad (6)$$

Experiments at LEP1, SLC, LEP2, and those planned at a linear collider aim at precisions well below a per cent and need theoretical predictions with an accuracy of the order of 0.1 %

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or better. A basic ingredient of the predictions is the complete photonic $O(\alpha)$ correction including initial and final state radiations and their interferences:

$$\sigma(s) = \sigma^0(s) + \sigma^{ini}(s) + \sigma^{int}(s) + \sigma^{fin}(s). \quad (7)$$

These corrections have to be determined for two basic quantities: The total cross-section $\sigma_T(s)$ and the forward-backward asymmetry $A_{FB} = \sigma_{F-B}/\sigma_T$; other asymmetries may then easily be derived.

Basically, there are two experimental set-up's to be treated:

- (i) a lower cut on s' , $s'_{min} \geq 4m_f^2$, often applied to quark-pair production;
- (ii) combined cuts on acollinearity ξ , $\xi_{max} \leq 180^\circ$, and minimal energy E_{min} , $E_{min} \geq m_f$, of the fermions; often applied to lepton-pair production.

Both cut settings may be combined with an acceptance cut c , $c \leq 1$, on the cosine of the fermionic production angle $\cos \theta$.

For case (i), a generalization of the Bonneau-Martin formula, including the complete first-order photonic corrections together with soft-photon exponentiation, may be found in [3] without and in [4] with acceptance cut. The extremely compact expressions for case (i) with $c = 1$ get quite more involved when the acceptance cut is applied.

In this article, we give the complete first-order photonic corrections together with soft-photon exponentiation for case (ii).

A three-fold analytical integration of the squared matrix elements is performed in order to get the integrand of the last one, the s' -integration over R , which is assumed to be performed numerically. One may use the phase-space parameterization derived in [5]. The kinematical regions of two variables are shown in figure 1: the (normalized) invariant mass x of (photon + anti-fermion) in their rest system, and R as introduced above. The first and third analytical integrations are over the full production angle of the photon, ϕ_γ , in the (photon + fermion) rest system, and over the cosine of the anti-fermion in the center-of-mass system, $\cos \vartheta$. Both angles are completely independent of x and R . As figure 1 shows, we have to determine radiators ρ_B^b ($B = T, F-B$ and $b = ini, int, fin$) in three phase-space regions:

$$\sigma_B^b(s) = \left[\int_I + \int_{II} - \int_{III} \right] d\phi_\gamma dx ds' d\cos \vartheta \frac{d\sigma_B^b(A)}{d\phi_\gamma dx ds' d\cos \vartheta}, \quad (8)$$

Region I applies to the simple s' -cut. The integration over R extends from R_{min} to 1,

$$R_{min} = R_E \left(1 - \frac{\sin^2(\xi_{max}/2)}{1 - R_E \cos^2(\xi_{max}/2)} \right). \quad (9)$$

In each of the three regions, the boundaries for the integration over x are, for a given value of R :

$$x_{max,min} = \frac{1}{2}(1 - R) (1 \pm A), \quad (10)$$

where the parameter $A = A(R)$ depends in every region on only one of the cuts applied:

$$A_I = \sqrt{1 - \frac{R_m}{R}}, \quad (11)$$

$$A_{II} = \frac{1 + R - 2R_E}{1 - R}, \quad (12)$$

$$A_{III} = \sqrt{1 - \frac{R(1 - R_\xi)^2}{R_\xi(1 - R)^2}}, \quad (13)$$

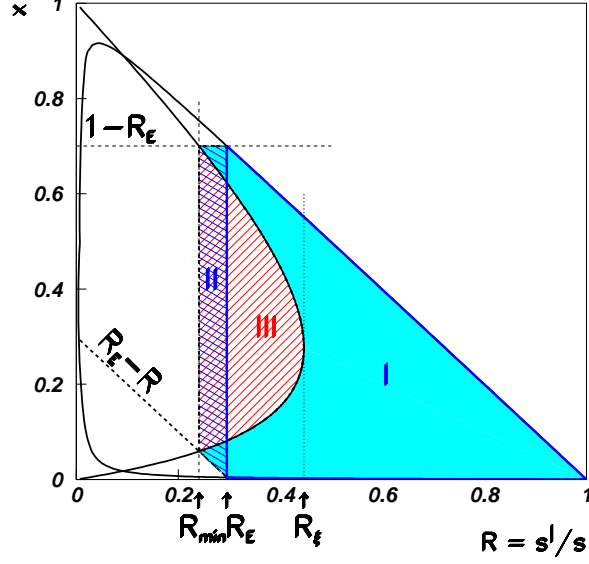


Figure 1: Phase space with cuts on acollinearity and (lower) fermion energy.

with:

$$R_m = \frac{4m_f^2}{s}, \quad (14)$$

$$R_E = \frac{2E_{min}}{\sqrt{s}}, \quad (15)$$

$$R_\xi = \frac{1 - \sin(\xi_{max}/2)}{1 + \sin(\xi_{max}/2)}. \quad (16)$$

Initial State Radiation

Here, for $\sigma_T^{ini}(s)$ the Bonneau-Martin function gets replaced by:

$$H_T^{ini}(R, A) = \frac{3\alpha}{4\pi} Q_e^2 \left[\left(A + \frac{A^3}{3} \right) \frac{1+R^2}{1-R} \left(\ln \frac{s}{m_e^2} - 1 \right) + (A - A^3) \frac{2R}{1-R} \right]. \quad (17)$$

In $\sigma_{F-B}^{ini}(s)$, the corresponding hard radiator part is:

$$\begin{aligned} H_{F-B}^{ini}(R; A \geq A_0) = & \frac{\alpha}{\pi} Q_e^2 \left\{ \frac{1+R^2}{1-R} \left[\frac{4R}{(1+R)^2} \left(\ln \frac{s(1+R)^2}{4m_e^2 R} - 1 \right) \right. \right. \\ & - \frac{1}{(1+R)^2} [y_+ y_- \ln |y_+ y_-| + 4R \ln(4R)] \\ & \left. \left. - (1-A^2) \left(\ln \frac{s}{4m_e^2 (1+A)^2 R} - 1 \right) \right] + \frac{4A(1-A)R}{1-R} \right\}, \quad (18) \end{aligned}$$

$$\begin{aligned} H_{F-B}^{ini}(R; A < A_0) = & \frac{\alpha}{\pi} Q_e^2 \left\{ \frac{1+R^2}{1-R} \left[-\frac{y_+ y_-}{(1+R)^2} \ln \left| \frac{y_+}{y_-} \right| + (1-A^2) \ln \frac{1+A}{1-A} \right] \right. \\ & \left. + \frac{8AR^2}{(1+R)(1-R)} \right\}. \quad (19) \end{aligned}$$

These functions have to be used in (3) and its analogue, with equal \bar{S} , for σ_{F-B}^{ini} . The following definitions are used:

$$A_0 = \frac{1-R}{1+R}, \quad (20)$$

$$y_{\pm} = (1-R) \pm A(1+R). \quad (21)$$

In region I, the above expressions (17) and (18) reduce to those known from [2] and [3]. In this region the radiators diverge for $R \rightarrow 1$, and soft-photon exponentiation and the subtraction $\beta/(1-R)$ is applied there (and only there) in order to get $\bar{H}_B^{ini}(R)$; see (5).

Initial-Final State Interferences

In the initial-final state interferences, the effective Born cross-sections depend on both s and s' as well as on the type of exchanged vector particles V_i (e.g. photon and or Z):

$$\sigma_B^{int}(s) = \int dR \sum_{V_i, V_j=\gamma, Z} \sigma_B^0(s, s', i, j) \rho_B^{int}(R, A, i, j). \quad (22)$$

For $B=T$ it is $\bar{B}=F-B$ and vice versa. We give as examples simple model-independent Born expressions in order to fix the overall normalization:

$$\sigma_T^0(s, s') = \sum_{V_i, V_j=\gamma, Z} \sigma_T^0(s, s', i, j) = \frac{4\pi\alpha^2}{3s'} \mathcal{V}, \quad (23)$$

$$\sigma_{F-B}^0(s, s') = \sum_{V_i, V_j=\gamma, Z} \sigma_{F-B}^0(s, s', i, j) = \frac{\pi\alpha^2}{s'} \mathcal{A}, \quad (24)$$

with

$$\mathcal{V} = Q_e^2 Q_f^2 + |Q_e Q_f| v_e v_f \Re[\chi(s) + \chi(s')] + (v_e^2 + a_e^2)(v_f^2 + a_f^2) \Re[\chi(s) \chi^*(s')], \quad (25)$$

$$\mathcal{A} = |Q_e Q_f| a_e a_f \Re[\chi(s) + \chi(s')] + 4v_e a_e v_f a_f \Re[\chi(s) \chi^*(s')], \quad (26)$$

and with the following Z boson propagators:

$$\chi(s) = \frac{G_\mu M_Z^2}{8\sqrt{2}\pi\alpha} \frac{s}{s - M_Z^2 + iM_Z\Gamma_Z}. \quad (27)$$

The radiator functions are:

$$\rho_B^{int}(R, A; i, j) = \delta(1-R) [S_B + b_B(i, j)] + \theta(1-R-\epsilon) H_B^{int}(R, A). \quad (28)$$

The box contributions $b_T(i, j)$ may be taken from equations (116) and (118) (to be multiplied by 4/3) of [6] and the $b_{F-B}(i, j)$ from equations (123) and (126). For convenience, we give the soft corrections explicitly:

$$S_T^{int} = 8\frac{\alpha}{\pi} Q_e Q_f \left(1 - \ln \frac{2\epsilon}{\lambda}\right), \quad (29)$$

$$S_{F-B}^{int} = \frac{\alpha}{\pi} Q_e Q_f \left[-(1 + 8\ln 2) \ln \frac{2\epsilon}{\lambda} + 4\ln^2 2 + \ln 2 + \frac{1}{2} + \frac{1}{3}\pi^2 \right]. \quad (30)$$

Finally, the hard radiator parts are:

$$H_T^{int}(R, A) = -\frac{\alpha}{\pi} Q_e Q_f \frac{4AR(1+R)}{1-R}, \quad (31)$$

and

$$\begin{aligned}
H_{F-B}^{int}(R, A \geq A_0) = & \frac{\alpha}{\pi} Q_e Q_f \left\{ \frac{3R}{2} \left[\ln \frac{z_+}{z_-} + \frac{2-R+\frac{5}{3}R^2}{1-R} \ln R \right] \right. \\
& - \frac{1+R}{2(1-R)} (5-2R+5R^2) \ln \frac{(1+R)(1+A)}{2} \\
& + \frac{1}{4(1-R)} \left[\frac{(1-4R+R^2)[A(1+R)^2-(1-R)^2]}{1+R} \right. \\
& \left. \left. + 2A(1-A)(1+R^3) \right] \right\}, \quad (32)
\end{aligned}$$

$$H_{F-B}^{int}(R, A < A_0) = \frac{3\alpha}{2\pi} Q_e Q_f R \left\{ \ln \frac{z_+}{z_-} - \frac{2-R+\frac{5}{3}R^2}{1-R} \ln \frac{1+A}{1-A} + A(1-R) \right\}, \quad (33)$$

with

$$z_{\pm} = (1+R) \pm A(1-R). \quad (34)$$

Again, for $A \rightarrow 1$ the $H_T^{int}(R, A)$ and $H_{F-B}^{int}(R, A \geq A_0)$ approach the known expressions of the s' cut given in [3]^b.

Final State Radiation

The final state corrections to order $O(\alpha)$ are:

$$\sigma_B^{fin}(s) = \sigma_B^0(s) \int dR \rho_B^{fin}(R, A), \quad (35)$$

with

$$\rho_B^{fin}(R, A) = \delta(1-R) S_f + \theta(1-R-\epsilon) H_B^{fin}(R, s, A), \quad (36)$$

$$S_f = \bar{S}_f + \beta_f \ln \epsilon, \quad (37)$$

where \bar{S}_f and β_f are the final state's analogues of \bar{S} and β . The hard radiators are:

$$H_T^{fin}(R, s, A) = \frac{\alpha}{\pi} Q_f^2 \left[\frac{1+R^2}{1-R} \ln \frac{1+A}{1-A} - \frac{8Am_f^2/s}{(1-A^2)(1-R)} - A(1-R) \right], \quad (38)$$

$$H_{F-B}^{fin}(R, s, A) = H_T^{fin}(R, A) + \frac{\alpha}{\pi} Q_f^2 \left[A(1-R) - (1+R) \ln \frac{z_+}{z_-} \right]. \quad (39)$$

Common initial- and final state soft-photon exponentiation may be performed as follows [7,6]:

$$\sigma_B^{ini+fin}(s) = \int dR \sigma^0(s') \rho_B^{ini}(R, A) \bar{\rho}_B^{fin}(R, s', A), \quad (40)$$

with

$$\bar{\rho}_B^{fin}(R, s', A) = (1-R_E)^{\beta'_f} (1+S'_f) + \int_{R_{min}/R}^1 du \left[H_B^{fin}(u, s', A') - \frac{\beta'_f}{1-u} \theta(R-R_E) \right]. \quad (41)$$

The soft part of $\rho_B^{fin}(u, s', A')$, $A' = A(u)$, and β'_f are derived from (3) by replacing there Q_e by Q_f and s/m_e^2 by s'/m_f^2 . We mention here that in the hard radiators the integration over u may also be performed analytically. In region III, one has to interchange for this the order of integration over u and x [8].

^b We realized a misprint in eq. (22) of [3]; the non-logarithmic terms have to be multiplied there by $1/(1+R)$.

Conclusions

We recalculated the photonic corrections with acollinearity cut having applications in the Fortran program **ZFITTER** in mind [9]. When the code was created in 1989 [10], an accuracy of 0.5 % at LEP1 was assumed to be needed ^c.

We performed several numerical applications of the above formulae. For this purpose, the package **acol.f** was added. As a result, we conclude that **ZFITTER** until version 5.20 treats the $O(\alpha)$ photonic corrections with acollinearity cut with a numerical accuracy for σ_T of not less than about 0.4 % near the Z resonance (LEP1 energy region, \sqrt{s} within $M_Z \pm 3$ GeV) or better (at resonance). The coding with **acol.f** gave a numerical agreement of σ_T for leptons, with $\xi_{max} \leq 10^\circ$ and $E_{min} = 1$ GeV, with predictions from **TOPAZ0** v.4.4 [13] at LEP1 of 0.01 % (at the wings) or better (at resonance) [14]. For A_{FB} , we estimate at LEP1 energies the accuracy of the $O(\alpha)$ photonic corrections with acollinearity cut in **ZFITTER** until v.5.20 to be about 0.02 % or better at the resonance and about 0.13 % or better at the wings. The numerical limitations at LEP1 are due to the initial-final state interference.

For applications at higher energies, the accuracy of **ZFITTER** with acollinearity cut was not dedicatedly controlled until recently, although there was reason to suspect that it comes out much worse than at LEP1. With the cuts mentioned, the accuracy of v.5.20 at LEP2 is again limited by the initial-final interference but not less than roughly 1 %.

A higher accuracy in the acollinearity mode is prevented in **ZFITTER** until v.5.20 by several reasons. The main reason is a neglect of a certain class of ordinary, angular dependent $O(\alpha)$ terms in the initial state and in the initial-final state interference hard radiator parts. Some numerical approximations in the treatment of final state radiation may also be influential; this has not been studied in detail so far.

Finally we should like to mention that the correct $O(\alpha)$ hard radiators for the angular distributions and for the integrated cross-sections with cuts on both acollinearity and angular acceptance also have been determined within this project and will be published elsewhere together with a sketch of the calculations and more numerical results.

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^c For other recent comparisons see e.g. [11,12] and for the influence of higher order corrections e.g. [12], and references therein.

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